Unit 7 Summary

Prior Learning	Grade 8, Unit 7	High School
Grade 6 • Performing operations with whole number exponents (e.g., using the formula $V = s^3$)	• Generate equivalent expressions involving positive exponents, zero exponents, and negative exponents. • Write numbers in scientific notation. • Estimate and perform operations with numbers in scientific notation.	• Write and analyze functions that use exponents. • Work with fractions as exponents.

Exponent Properties

$8^6 = 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$

Base = 8

Exponent = 6

Power = 8^6

Exponents are a way of keeping track of how many times a number has been repeatedly multiplied.

Using our understanding of repeated multiplication, we can figure out several properties of exponents.

$10^3 \cdot 10^4 = (10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10 \cdot 10)$

$10^3 \cdot 10^4 = 10^7$

Another way to multiply powers with the same base is to add their exponents together.

$(7^2)^3 = (7 \cdot 7) \cdot (7 \cdot 7) \cdot (7 \cdot 7)$

$(7^2)^3 = 7^6$

Another way to express powers of powers can be found by multiplying the exponents together.

$10^{-3} = \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} = \frac{1}{10 \cdot 10 \cdot 10} = \frac{1}{10^3}$

$8^0 = 1$

Negative exponents and zero exponents extend from the properties of positive exponents.

desmos

Unit 8.7, Family Resource

Scientific Notation

The United States mint has made over $500,000,000,000$ pennies.

A single carbon atom weighs $0.00000000000000000000002$ grams.

The distance from Earth to the moon is $240,000$ miles.

$$500,000,000,000 = 5 \cdot 10^{11}$$

$$0.00000000000000000000002 = 2 \cdot 10^{-23}$$

Another way to write very large and very small numbers is as multiples of powers of 10.

Writing numbers in this way helps avoid errors since it would be easy to accidentally add or take away a zero when writing out the decimal.

$240\,000 = 2 \cdot 10^5 + 4 \cdot 10^4$ or $24 \cdot 10^4$

Scientific notation: $2.4 \cdot 10^5$

Scientific notation is one specific way to write numbers.

Numbers in scientific notation are written as a number between 1 and 10 multiplied by a power of 10.

$$2.4 \cdot 10^5 < 1.5 \cdot 10^6$$

It is more efficient to compare numbers when they are both written in this form.

Try This at Home

Exponent Properties

1.1 Carlos and Amara were trying to understand the expression $3^4 \cdot 3^5$. Amara said, “Since we are multiplying, we will get 3^{20}.” Carlos said, “But I don’t think you can get 3 twenty times by multiplying everything together.” Do you agree with either of them?

1.2 Next, Carlos and Amara were thinking about the expression $(3^4)^5$. Amara said, “Okay, this one will be 3^{20} because you will have five groups of four 3s.” Carlos said, “I agree it will be 3^{20}, but it’s because there will be four groups of five 3s.” Do you agree with either of them?

Scientific Notation

This table shows the top speeds of different vehicles.

Vehicle	Speed (kilometers per hour)
Sports car	$4.15 \cdot 10^2$
Apollo command and service module (Mother ship of the Apollo spacecraft)	$3.99 \cdot 10^4$
Jet boat	$5.1 \cdot 10^2$
Autonomous drone	$2.1 \cdot 10^4$

2.1 Order the vehicles from fastest to slowest.

2.2 The top speed of a rocket sled is $10,326$ kilometers per hour. Is this faster or slower than the autonomous drone?

2.3 Estimate how many times as fast the Apollo command and service module is as the sports car.

Solutions:

1.1 Carlos is correct. Rewriting $3^4 \cdot 3^5$ to show all the factors looks like:

$$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$$

We can see that there are a total of 3s multiplied together nine times. This helps us understand what's going on when we use the rule to write $3^4 \cdot 3^5 = 3^{4+5} = 3^9$.

1.2 This time, Amara is correct. When we look at $(3^4)^5$, the outside exponent of 5 tells us that there are five factors of 3^4 being multiplied together. So $(3^4)^5 = 3^4 \cdot 3^4 \cdot 3^4 \cdot 3^4 \cdot 3^4$. We could write this out the long way as:

$$(3^4)^5 = (3 \cdot 3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3).$$

This helps us understand what's going on when we use the rule to write $(3^4)^5 = 3^{4 \cdot 5} = 3^{20}$.

2.1 The order from fastest to slowest is:

- Apollo CSM
- Autonomous drone
- Jet boat
- Sports car

Since all of these values are in scientific notation, we can look at the power of 10 to compare. Both the speeds of the Apollo CSM and the autonomous drone have the highest power of ten, 10^4, so they are the fastest. The Apollo CSM is faster than the drone because 3.99 is greater than 2.1. Similarly, the jet boat is faster than the sports car because 5.1 is greater than 4.15, even if their speeds both have the same power of ten, 10^2.

2.2 The drone is faster than the rocket sled. One approach is to convert the rocket sled's speed into scientific notation. $10,326$ is equivalent to $1032.6 \cdot 10$, which is equivalent to $103.26 \cdot 10 \cdot 10$. By continuing that process, we can determine that the rocket sled's speed is $1.0326 \cdot 10^4$. The drone's speed is $2.1 \cdot 10^4$ kilometers per hour and 2.1 is greater than 1.0326, so the drone must be faster.

2.3 To compare the speeds of the Apollo CSM and the sports car, we can try to find the missing number in this equation: $? \cdot 4.15 \cdot 10^2 = 3.99 \cdot 10^4$. To find the missing value (?), we need to compute $\frac{3.99 \cdot 10^4}{4.15 \cdot 10^2}$. Since we are estimating, we can simplify the calculation to $\frac{4 \cdot 10^4}{4 \cdot 10^2}$. Using properties of exponents and our understanding of fractions, we can conclude that $\frac{4 \cdot 10^4}{4 \cdot 10^2} = 1 \cdot 10^2$, so the Apollo CSM is about 100 times as fast as the sports car!